Name ________________________________ Period __________ Date ______________________

14 Months

The Situation: You've just won the new lottery game called 14 Months. You have three options as to how you can choose to receive your prize money for the next 14 months. Read the options below and then make a selection just based on your intuition (your general feeling) without doing any calculations.

Option A	Option B	Option C
Receive $5000 right now plus get $1000 each month for the next 14 months.	Receive nothing right now but get $100 on month 1, $300 on month 2, $500 on month 3, $700 on month 4, etc. Each month the amount goes up by $200. This continues for 14 months.	Receive $1 right now and get $1 on month 1, $2 on month 2, $4 on month 3, $8 on month 4, etc. Each month doubling the previous month. This continues for 14 months.

1. Which option seems best to you at first glance? ______________________________

2. What makes this option appear attractive? ______________________________
   Discuss this with your teammates and see if you agree or disagree about the best option.

Analyzing the First Seven Months

Complete the tables to find the total amount of money you would receive during the first 7 months.

Option A

Month	Total $
1	$6000
2	$7000
3	
4	
5	
6	
7	

Option B

Month	Total $
1	$100
2	
3	
4	
5	
6	
7	

Option C

Month	Total $
1	
2	
3	
4	
5	
6	
7	

3. Plot the totals by month for the three options on https://www.desmos.com/calculator/tgsn7eoerg

4. Describe what you notice about each of the plots.

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5. Write an expression that will give the total for each of the options on the x^{th} month.

A(x) = ____________________ B(x) = ____________________ C(x) = ____________________

6. Enter these functions below the tables on the Desmos graph. Confirm that they match your data points.

Reevaluating Your Choice

7. Now that you have seen the total for the first 7 months and can see the progress on the graph, how confident are you about your initial choice of the best option? Explain and give a new best choice if necessary.

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Using Graphs and Using Equations to Solve

8. Use the space below to sketch how the graphs can be used to find when each function reaches $10,000. Label these points on your sketch.

9. Use the space below to show how to use the equations to find when each function reaches $10,000.

10. Above you solved three equations graphically and algebraically. Discuss with your teammates the advantages and disadvantages of each method.

Analyzing the Next 7 Months

11. Complete the tables to find the total amount of money you would receive during the next 7 months.

Option A

Month	Total $
8	
9	
10	
11	
12	
13	
14	

Option B

Month	Total $
8	
9	
10	
11	
12	
13	
14	

Option C

Month	Total $
8	
9	
10	
11	
12	
13	
14	

12. Describe your observations.

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Using Graphs and Using Equations to Solve Again

13. Use the space below to sketch how the graphs can be used to find when $A(x) = B(x)$ and when $B(x) = C(x)$. Label the intersections on your sketch.

14. Use the space below to show how to use the equations to find when $A(x) = B(x)$ and when $B(x) = C(x)$.

15. Again you solved equations two ways. Discuss with your teammates if some equations cannot be solved graphically, or cannot be solved algebraically.

Final Analysis

16. Which option gives the highest total for the 14 months? ____

17. If the money was given using the same functions but for a different number of months, then the other two options might have given the highest total. Explain clearly using complete sentences which number of months would make each of the other two options give the highest total.

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18. Why would the type of function that defines Option C always give the greatest total for very large numbers of months?

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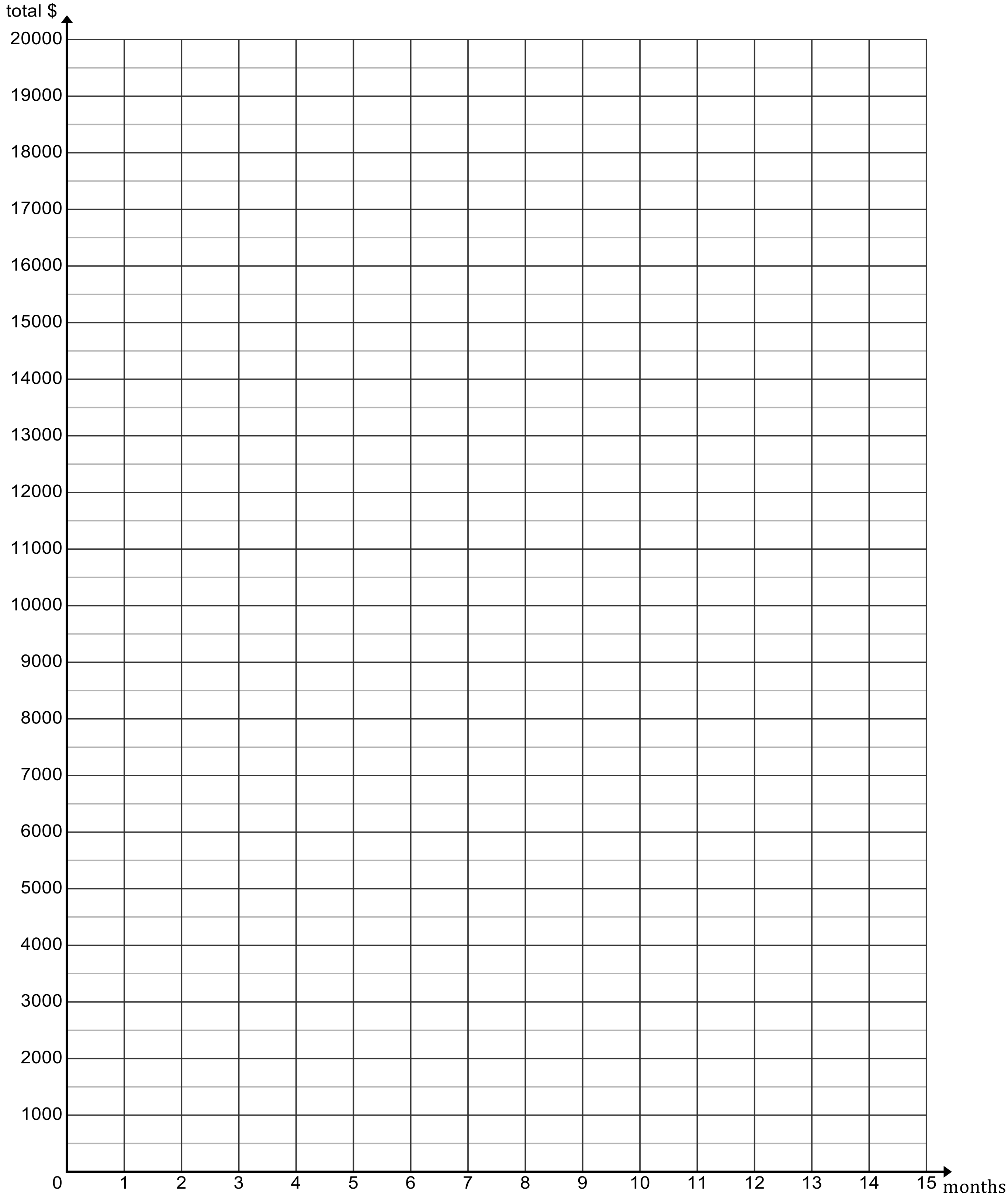
Extension

19. Use both the graphs and the equations to find how long it takes for each function to reach 1 million dollars. Give your answer in years and months.

14 Months Lottery Options Graph

Be sure to use three colors and label each graph with its option letter and its function.



Teacher Directions: 14 Months

Materials: Access to Desmos. Colored pencils if the paper graph is used.

Objective: Students will:

- Make predictions about the growth of number patterns
- Write expressions to match data tables
- Write and solve equations algebraically and using graphs.
- Compare and contrast algebraic and graphical methods for solving equations.

Directions:
This is primarily a team activity (teams of 3 or 4 are best) with several opportunities for whole class discussion, argument presentation, and critiquing the arguments of others. This activity also serves as a review of linear, quadratic and exponential functions.

There should be a back and forth motion of activity shifting between the teams and their discussions and whole class moderated discussions. It isn't necessary for all teams to be at the same place for a class discussion, but it is better if most teams have completed and discussed among themselves a section before the whole class talks about it.

Pass out *14 Months* and have a volunteer read *The Situation* and the three options. Briefly discuss each option with the class, making sure students understand how each of the three options will pay out the lottery winnings. Have students individually complete the first two questions, and then share with their group. The first whole class discussion moment should happen after students have selected which option appears at first glance to be best to them. **Use your questions to focus on how each option begins and how the payouts change over the months.** Make sure different viewpoints are shared but refrain from giving your opinion about each statement. Then, have students move on to completing the first set of tables and entering the data into Desmos. In teams, students should discuss what they notice about each of the patterns and record their observations on page 1. Students will then work together to write an expression for each of the three functions.

Note: If groups struggle with finding the equation for the functions, here are some questions you might pose, first general ones and then specifically for each option:

- Ask students to look at the graph and table and then ask, "What pattern(s) do you notice? How is the table growing? How is the graph growing?"
- Does this look like a function you have worked with in the past? What type?
- If you recorded the 'received this month' amount, where would it go in the table, and what does it represent? Where do you see this amount on the graph? What does this value represent in the equation?

Specific Questions for Option A:

- What do you notice about the rate of change?
- What do we know about functions that have a constant rate of change?
- Have students physically write in the rate of change, either on the table (as shown to the right), or on the graph, using slope triangles.

Option A

Month	Total $
1	$6000
2	$7000
3	$8000
4	$9000
5	$10000
6	$11000
7	$12000

+1000
+1000
+1000
+1000

Specific Questions for Option B:

- Is there a constant rate of change? If so, what is it? If not, is there a constant rate of change for the second difference, or third? Is there a constant multiplier (or common ratio)?
- What do we know about a function with a constant second difference?
- What type of relationship do you see between the day and the total dollar amount?

Option B – Second Common Difference

Month	Total $
1	$100
2	$400
3	$900
4	$1600
5	$2500
6	$3600
7	$4900

+300
+500
+700
+900

+200
+200
+200

A constant second difference indicates that the data represent a quadratic function.

To assist students with writing the equation, we can ask if they can rewrite the total in another way, in order to look for a relationship between the day and total dollar amount.

Option B

Month	Total $
1	$100 = 1 (100)
2	$400 = 4 (100)
3	$900 = 9 (100)
4	$1600 = 16 (100)
5	$2500 = 25 (100)
6	$3600 = 36 (100)
7	$4900 = 49 (100)

Using the pattern, we can see that the total is the square of the month times 100; Total = $\text{month}^2(100)$. Therefore, the equation is $B(x) = 100\,(x^2)$.

Specific Questions for Option C

- Is there a constant rate of change? If so, what is it? If not, is there a constant rate of change for the second difference, or third? Is there a constant multiplier (or common ratio)?
- What do we know about a function with a constant multiplier (or common ratio)?
- What type of relationship do you see between the day and the total dollar amount?

Option C – Multiplicative Rate of Change

Month	Total $	
1	\$2	x2
2	\$4	x2
3	\$8	x2
4	\$16	x2
5	\$32	
6	\$64	
7	\$128	

A constant multiplier or common ratio indicates the data represent an exponential function.

To assist students with writing the equation, we can ask if they can rewrite the total in another way, in order to look for a relationship between the month and total dollar amount.

Option C – Multiplicative Rate of Change

Month	Total $
1	$\$2 = 2^1$
2	$\$4 = (2)(2)$ or 2^2
3	$\$8 = (2)(2)(2)$ or 2^3
4	$\$16 = (2)(2)(2)(2)$ or 2^4
5	\$32
6	\$64
7	\$128

Using the pattern, we can see that the total is 2 raised to the month; Total = 2^{month}. Therefore, the equation is $C(x) = 2^x$.

The second whole class discussion should happen after most teams have finished analyzing the first seven months and written expressions for each option. **Use your questions to focus on the match (or lack of match) between the expressions, the tables, the graphs and the description of the payout option. Also draw attention to what type of function they have found matches each option.**

Have students move on to page 2. The original window on the Desmos graph does not show past 8 months, but many students may quickly drag or scroll to see the next 7 months (ideally they will wait, though!). The primary emphasis of page 2 is to look at solving equations graphically and algebraically, noticing the strengths and challenges of each method.

The third whole class discussion should happen after most teams have finished the entire second page. **Use your questions to draw attention to the core meaning of solving equations. We can find the values that satisfy an equation by considering the functions from which they arise and using the associated graphs, or we can apply algebraic techniques to the equations themselves.**

The third page examines the second half of the payment time period and brings up the question of when the payment options have equal totals. The equations that arise out of this question are more complicated than the equations on page 2. The first of these equations has a linear expression equal to a quadratic expression.

$$1000x + 5000 = 100x^2$$

This can be solved graphically by looking at the intersection of the linear Option A with the quadratic Option B, or by factoring or quadratic formula and solving algebraically.

The second of these equations has a quadratic expression equal to an exponential expression.

$$100x^2 = 2^x$$

This can be solved graphically but cannot be solved using our usual algebraic processes of applying inverse operations. It is important to note that the inability to solve the equation using inverses does not imply that there isn't a solution, but rather draws attention to the value of other approaches to solving (namely, using the graphs of the functions).

The fourth page brings us back to the question of payment options. While there is a clear answer to which option will give the largest total amount by the end of the 14 months, it is reasonable that some students might argue that they would prefer Option A, which is only $600 less than Option B but provides much larger totals at the start of the payment period.

This page also draws attention to the question of which payment option is ahead at what number of months and the long-run dominance of exponential functions.